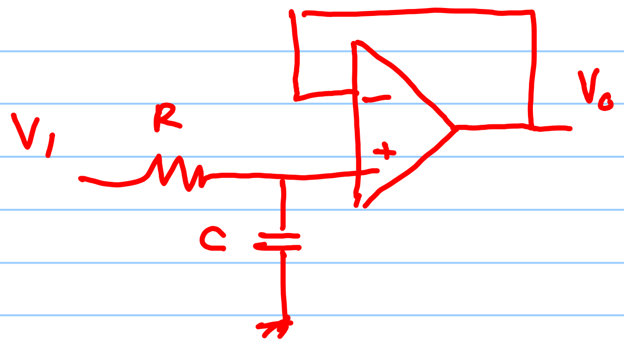
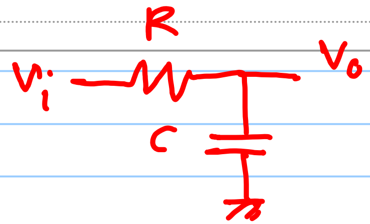
Desain Filter Aktif



$\Rightarrow$



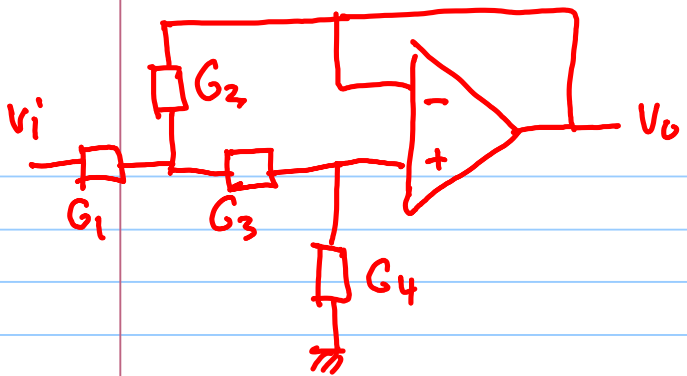
$$\frac{V_o}{V_i} = \frac{\frac{1}{sC}}{\frac{1}{sC} + R} = \frac{1}{1 + sCR} = \frac{1}{1 + \frac{s}{1/CR}}$$

$$\omega_o = \frac{1}{CR} \quad \text{atau}$$

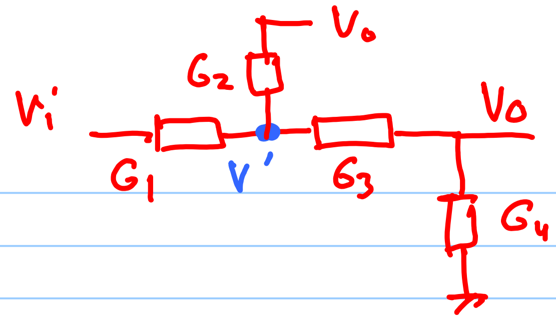
$$\frac{V_o}{V_i} = \frac{1}{1 + sCR} = \frac{1/RC}{s + 1/RC}$$

$$\omega_o = \frac{1}{RC} \Rightarrow \frac{V_o}{V_i} = \frac{\omega_o}{s + \omega_o}$$

Normalisasi: $\omega_o = 1 \Rightarrow \frac{V_o}{V_i} = \frac{1}{s+1}$ $\leftarrow$ orde 1



$\Rightarrow$



$$V_o = \frac{\frac{1}{G_4}}{\frac{1}{G_4}+\frac{1}{G_3}} \cdot V'$$

$$V_o = \frac{G_3}{G_3+G_4} V'$$

$$V' = \frac{G_3+G_4}{G_3} V_o \quad \dots (1)$$

Node V':

$$(V'-V_i)G_1 + (V'-V_o)G_3 + (V'-V_o)G_2 = 0$$

$$V'(G_1+G_2+G_3) = V_i G_1 + V_o(G_2+G_3) \quad \dots (2)$$

Substitusi (1) & (2):

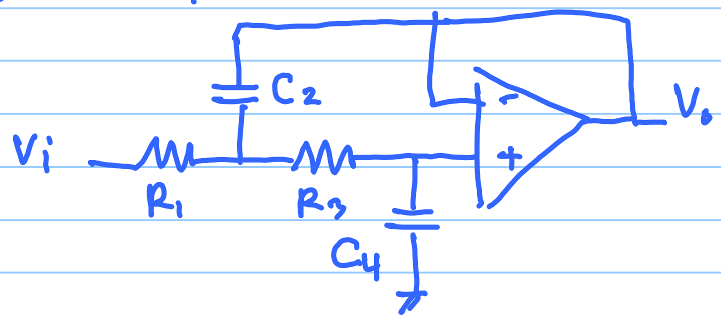
$$V_o \frac{G_3+G_4}{G_3}(G_1+G_2+G_3) = V_i G_1 + V_o(G_2+G_3)$$

$$V_o(G_3+G_4)(G_1+G_2+G_3) = V_i G_1 G_3 + V_o G_3 (G_2+G_3)$$

$$V_o\left[\left(G_3G_1 + \cancel{G_3G_2} + \cancel{G_3^2} + G_4G_1 + G_4C_2 + G_4G_3\right) - \left(\cancel{G_3G_2} + \cancel{G_3^2}\right)\right] = V_i G_1 G_3$$

$$\boxed{\frac{V_o}{V_i} = \frac{G_1G_3}{G_4G_1 + G_4G_2 + G_4G_3 + G_1G_3}}$$

Contoh : LPF



$$\frac{V_o}{V_i} = \frac{G_1G_3}{G_4G_1 + G_4G_2 + G_4G_3 + G_1G_3}$$

$$= \frac{\frac{1}{R_1}\cdot\frac{1}{R_3}}{sC_4\frac{1}{R_1} + sC_4 sC_2 + sC_4\frac{1}{R_3} + \frac{1}{R_1}\cdot\frac{1}{R_3}}$$

$$= \frac{1/R_1R_3}{s^2C_2C_4 + sC_4\left(\frac{1}{R_1} + \frac{1}{R_3}\right) + 1/R_1R_3}$$

$$\frac{V_o}{V_i} = \frac{\frac{1}{R_1R_3C_2C_4}}{s^2 + s\frac{1}{C_2}\left(\frac{1}{R_1}+\frac{1}{R_3}\right) + \frac{1}{R_1R_3C_2C_4}}$$

Normalisasi $\omega_o^2 = 1 = \frac{1}{C_2C_4}$

$$\frac{V_o}{V_i} = \frac{1}{s^2 + s\frac{2}{C_2} + 1}$$ ← orde 2

Perancangan nilainya :

Normalisasi R dalam 1Ω dan C dalam 1F

$R_1R_3C_2C_4 = 1$

Nilai $R_1 = R_3 = 1\Omega$, maka $C_2C_4 = 1$

$$\frac{1}{C_2}\left(\frac{1}{R_1}+\frac{1}{R_3}\right) = 1{,}414$$

Normalisasi thd $R_1 = R_3 = 1\Omega$

$$\frac{V_o}{V_i} = \frac{\frac{1}{C_2C_4}}{s^2 + s\frac{2}{C_2} + \frac{1}{C_2C_4}}$$

Butterworth Pole Locations

Order	Real -a	Imaginary +/-jb
2	0.7071	0.7071
3	0.5000	0.8660
	1.0000	

Normalized Butterworth

- The order of the filter is given by the normalized denominator of the transfer function known as normalized Butterworth polynomial.

Order of Filter	Butterworth Polynomial
1	$(s+1)$
2	$s^2+1.414s+1$
3	$(s+1)(s^2+s+1)$
4	$(s^2+0.765s+1)(s^2+1.848s+1)$
5	$(s+1)(s^2+0.618s+1)(s^2+1.618s+1)$

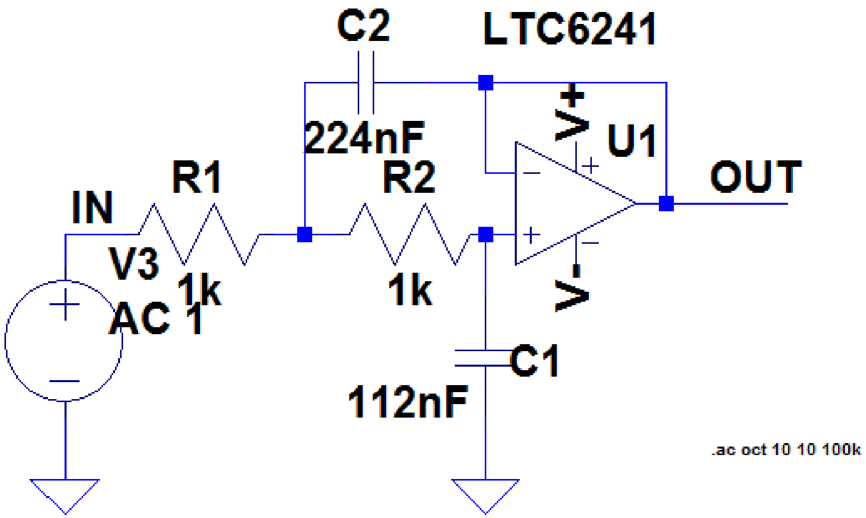
$$\Rightarrow \frac{2}{C_2} = 1{,}414 \Rightarrow C_2 = 1{,}418F$$

$$C_4 = \frac{1}{C_2} = 0{,}705F$$

Bila $R_1 = R_3 = 1k\Omega$, maka $C_2 = \frac{C_{2n}}{2\pi f \times R_1} = \frac{1{,}418}{2\pi . 1000 . 1000} = 226 \text{ nF}$

frekuensi cut off

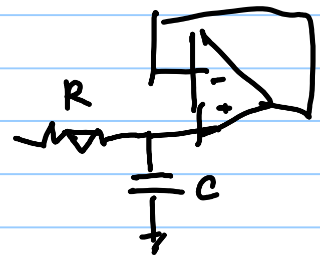
$C_4 = \frac{C_{4n}}{2\pi f \times R} = \frac{0{,}705}{2\pi . 1000 . 1000} = 114 \text{ nF}$



n (order)	Normalized Denominator Polynomials in Factored Form
1	(1+s)
2	$(1+1.414s+s^2)$
3	$(1+s)(1+s+s^2)$
4	$(1+0.765s+s^2)(1+1.848s+s^2)$
5	$(1+s)(1+0.618s+s^2)(1+1.618s+s^2)$
6	$(1+0.518s+s^2)(1+1.414s+s^2)(1+1.932s+s^2)$
7	$(1+s)(1+0.445s+s^2)(1+1.247s+s^2)(1+1.802s+s^2)$
8	$(1+0.390s+s^2)(1+1.111s+s^2)(1+1.663s+s^2)(1+1.962s+s^2)$
9	$(1+s)(1+0.347s+s^2)(1+s+s^2)(1+1.532s+s^2)(1+1.879s+s^2)$
10	$(1+0.313s+s^2)(1+0.908s+s^2)(1+1.414s+s^2)(1+1.782s+s^2)(1+1.975s+s^2)$

Filter orde 3 LPF

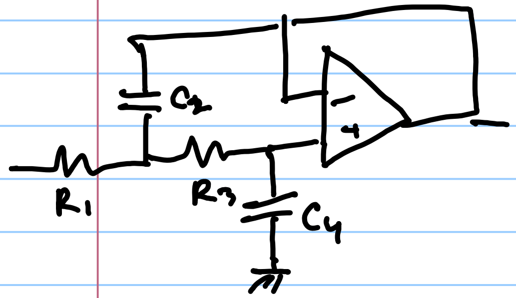
$$\underbrace{(s+1)}_{\text{orde 1}}\underbrace{(s^2+s+1)}_{\text{orde2}}$$



$\Rightarrow$ $\omega_0 = \frac{1}{RC} \Rightarrow \omega_0 = 1$

$RC = 1$

$R = 1\Omega \Rightarrow C = 1F$



$$\omega_0^2 = \frac{1}{R_1R_3C_2C_4} = 0$$

$R_1R_3C_2C_4 = 1$

$R_1 = R_3 = 1\Omega$

$C_2C_4 = 1$

$$\frac{1}{C_2}\left(\frac{1}{R_1}+\frac{1}{R_3}\right) = 1$$

$C_2 = 2F$

$C_4 = 0{,}5F$

sehingga :

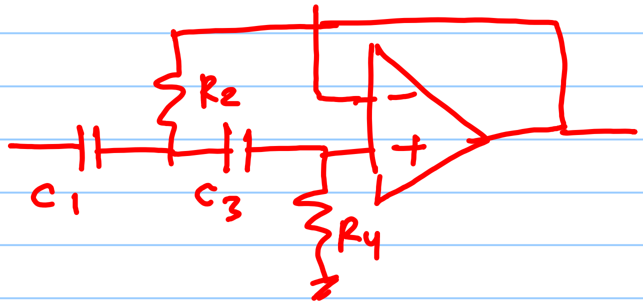
Jika $R = R_1 = R_3 = 1k$

$$C = \frac{C_n}{2\pi f . R} = \frac{1}{2\pi . 1000 . 1000} = 159{,}2\ nF$$

$$C_2 = \frac{C_{2n}}{2\pi f R} = \frac{2}{2\pi \cdot 1000 \cdot 1000} = 318{,}3 \text{ nF}$$

$$C_4 = \frac{C_{4n}}{2\pi f R} = \frac{0{,}5}{2\pi \cdot 1000 \cdot 1000} = 79{,}6 \text{ nF}$$

High Pass Filter (HPF)



Normalisasi thd $C_1 = C_3 = 1F$

$$\frac{V_o}{V_i} = \frac{G_1 G_3}{G_4 G_1 + G_4 G_2 + G_4 G_3 + G_1 G_3}$$

$$\frac{V_o}{V_i} = \frac{sC_1 \, sC_3}{\frac{1}{R_4} \cdot sC_1 + \frac{1}{R_4} \cdot \frac{1}{R_2} + \frac{1}{R_4} sC_3 + sC_1 sC_3}$$

$$\frac{V_o}{V_i} = \frac{s^2 C_1 C_3}{s^2 C_1 C_3 + s \cdot \frac{1}{R_4}(C_1 + C_3) + \frac{1}{R_4} \cdot \frac{1}{R_2}}$$

$$\frac{V_o}{V_i} = \frac{s^2}{s^2 + s \frac{2}{R_4} + \frac{1}{R_4} \cdot \frac{1}{R_2}}$$

$$\boxed{\frac{V_o}{V_i} = \frac{s^2 R_2 R_4}{s^2 R_2 R_4 + s2R_2 + 1}}$$

Butterworth Pole Locations

Order	Real -a	Imaginary +/-jb
2	0.7071	0.7071
3	0.5000	0.8660
	1.0000	

$$-0,7071 \pm j0,7071 = -(0,7071 \pm j0,707) \cdot \frac{(-0,7071 \pm j0,7071)}{(-0,7071 \pm j0,7071)}$$

$$= \frac{0,999}{-0,7071 \pm j0,7071}$$

Inverting $\Rightarrow \dfrac{-0,7071 \pm j0,7071}{0,999} = -0,7071 \pm j0,7071$

Poles

$$-0,7071 \pm j0,7071 \Rightarrow s^2 + 1,4014s + 1 \Rightarrow s^2 + 1,4945 + 1$$

$$2R_2 = 1,4014$$

$$R_2 = 0,7007$$

$$R_4 = \frac{1}{R_2} = \frac{1}{0,7007} = 1,427$$

$C_1 = C_3 = 1F$
$R_2 = 0{,}7007$
$R_4 = 1{,}4014$

Jika $C_1 = C_3 = 1nF$, maka $R_2 = \dfrac{R_{2n}}{2\pi f C} = \dfrac{0{,}7007}{2\pi \cdot 1000 \cdot 10^{-9}} = 111{,}5 k\Omega$

$R_4 = \dfrac{R_{4n}}{2\pi f C} = \dfrac{1{,}4014}{2\pi \cdot 1000 \cdot 10^{-9}} = 223 k\Omega$

Perancangan HPF Butterworth orde 3

Butterworth Pole Locations

Order	Real -a	Imaginary +/-jb
2	0.7071	0.7071
3	0.5000 1.0000	0.8660

Pole : $-0{,}500 \pm j0{,}866$ → double pole
$-1{,}000$ → single pole

$$\frac{V_o}{V_i} = \frac{s^2 R_2 R_4}{s^2 R_2 R_4 + s 2 R_2 + 1}$$

$-0{,}500 \pm j0{,}866 \Rightarrow -(0{,}500 \pm j0{,}866) \times \frac{(0{,}500 \pm j0{,}866)}{(0{,}500 \pm j0{,}866)}$

$= \frac{0{,}9999}{0{,}500 \pm j0{,}866} =$

inverting $\Rightarrow -0{,}5 \pm j0{,}866$

Double pole : $-0{,}5 \pm j0{,}866$

$(s + 0{,}5 + j0{,}866)(s + 0{,}5 - j0{,}866)$

$s^2 + s + 1$

Normalisasi $C_1 = C_3 = 1F$, maka : $2R_2 = 1 \Rightarrow R_2 = 0{,}5\,\Omega \Rightarrow R_2 R_4 = 1 \Rightarrow R_4 = 2\,\Omega$

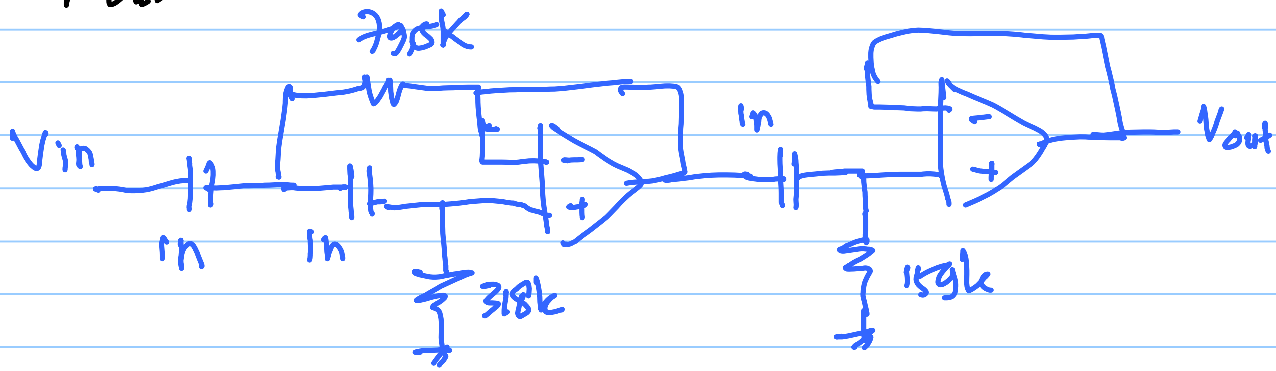
Sehingga jika $f = 1kHz$ dgn nilai $C_1 = C_3 = 1nF$

maka : $R_2 = \frac{R_{2n}}{2\pi f C} = \frac{0{,}5}{2\pi . 1000 . 10^{-9}} = 79{,}5\,k\Omega$

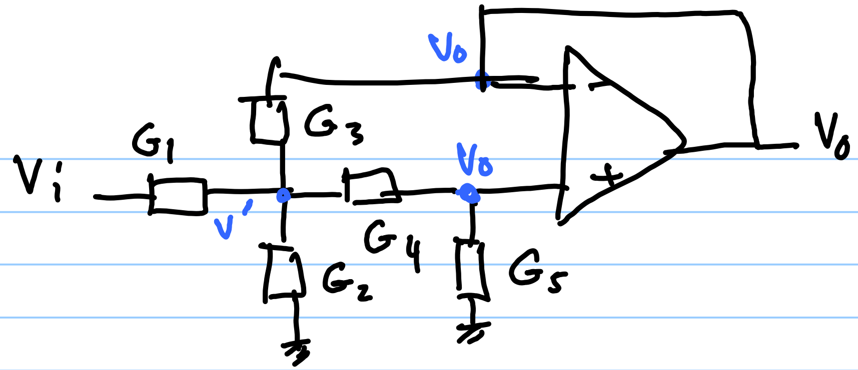
$R_4 = \frac{R_{4n}}{2\pi f} = \frac{2}{2\pi . 1000 . 10^{-9}} = 318\,k\Omega$

Single pole : -1,000

Moka :



Band Pass Filter



$$V_o = \frac{G_4}{G_4+G_5} \cdot V'$$

$$V' = \frac{G_4+G_5}{G_4} \cdot V_o$$

Node voltage V':

$$(V'-V_i)\,G_1 + (V'-V_o)\,G_3 + (V'-0)\,G_2 + (V'-V_o)\,G_4 = 0$$

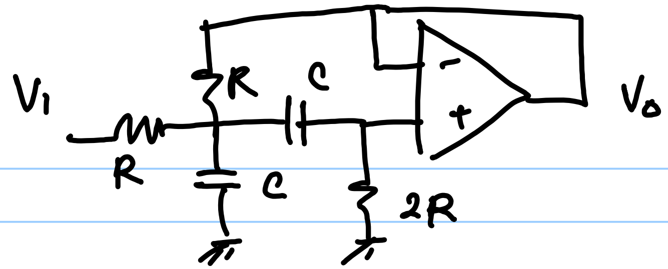
$$V'(G_1+G_2+G_3+G_4) - V_o(G_3+G_4) = V_i G_1$$

$$\frac{(G_4+G_5)}{G_4} \cdot V_o\,(G_1+G_2+G_3+G_4) - V_o(G_3+G_4) = V_i G_1$$

$$V_o\,(G_4G_1 + G_4G_2 + \cancel{G_4G_3} + \cancel{G_4^2} + G_5G_1 + G_5G_2 + G_5G_3 + G_5G_4 - \cancel{G_3G_4} - \cancel{G_4^2}) = V_i\,G_1G_4$$

$$\boxed{\frac{V_o}{V_i} = \frac{G_1G_4}{G_5(G_1+G_2+G_3+G_4) + G_4(G_1+G_2)}}$$

$$\frac{V_0}{V_i} = \frac{G_1G_4}{G_5(G_1+G_2+G_3+G_4) + G_4(G_1+G_2)}$$



$$\frac{V_0}{V_i} = \frac{\frac{1}{R}sC}{\frac{1}{2R}\left(\frac{1}{R}+sC+\frac{1}{R}+sC\right)+sC\left(\frac{1}{R}+sC\right)} = \frac{\frac{sC}{R}}{\frac{1}{2R}\left(\frac{2}{R}+2sC\right)+sC\left(\frac{1}{R}+sC\right)} = \frac{\frac{sC}{R}}{\frac{1}{R^2}+\frac{sC}{R}+\frac{sC}{R}+s^2C^2}$$

$$\frac{V_0}{V_i} = \frac{\frac{sC}{R}}{s^2C^2+2\frac{sC}{R}+\frac{1}{R^2}} = \frac{\cancel{C^2}}{\cancel{C^2}}\,\frac{\frac{s}{RC}}{s^2+\frac{2s}{RC}+\frac{1}{C^2R^2}} = \frac{s\cdot\frac{1}{RC}}{s^2+2s\cdot\frac{1}{RC}+\left(\frac{1}{RC}\right)^2}$$

dunano $\omega_0 = \frac{1}{RC}$ $\Rightarrow$ $\frac{V_0}{V_i} = \frac{\omega_0 s}{s^2+2s\omega_0+\omega_0^2}$ $\leftarrow$ ordo 2

BPF orde 2 Butterworth

$$\frac{V_o}{V_i} = \frac{s \cdot \frac{1}{RC}}{s^2 + 2s \cdot \frac{1}{RC} + \left(\frac{1}{RC}\right)^2} \leftarrow s^2 + 1{,}414\, s + 1 \Longrightarrow \frac{2}{RC} = 1{,}414$$

Jika $R = 1\Omega$, maka $C = \frac{2}{1{,}414} = 1{,}414F$

sehingga jika $R = 1k\Omega$ maka

$$C = \frac{1{,}414}{2\pi \cdot 1000 \cdot 1000} = 225nF$$